

# 12th Mini Workshop on Singularities, Geometry and Differential Equations 1st Meeting on Foliations and Singularities

January 28th - February 2nd, 2019 Federal University of Espírito Santo Vitória - Espírito Santo - Brazil

# Book of abstract

#### **Organizing Committee:**

Leonardo Câmara (UFES)
Leandro Nery de Oliveira (UFAC)
Maico Felipe Silva Ribeiro (UFES)
Maria Aparecida Soares Ruas (ICMC-USP)
Marcelo Saia (ICMC-USP)
Raimundo Araújo dos Santos (ICMC-USP)
Thiago Filipe da Silva (UFES)

#### **Scientific Committee:**

César Camacho (IMPA - Brazil)
Maurício Corrêa (UFMG - Brazil)
Thiago Fassarella (UFF - Brazil)
Alexandre Fernandes (UFC - Brazil)
Terence Gaffney (NEU - USA)
Miriam Manoel (ICMC-USP - Brazil)
Maria Ap. Soares Ruas (ICMC-USP - Brazil)
Marcelo Saia (ICMC-USP - Brazil)
José Seade (UNAM - Mexico)
Mihai Tibar (Lille 1 - France)













#### Introduction

The 12th Mini-Workshops on Real and Complex Singularities form a series of biennial meetings organized by the Singularities group of the Instituto de Ciências Matemáticas e de Computação of São Paulo University (ICMC-USP), located in the city of São Carlos, São Paulo state, Brazil.

Their main purpose is to bring together world experts and young researchers in singularity and foliation theories and related fields to report recent achievements and exchange ideas, addressing trends of researches in a stimulating environment.

The twelfth edition of the Mini-Workshop will be held at Federal University of Espirito Santo (UFES) from January 28th to February 1st 2019, and it with run together with the 1st Meeting on Foliations and Singularities organized by the group of Foliations and Singularities of UFES, Vitória city, Espirito Santo state.

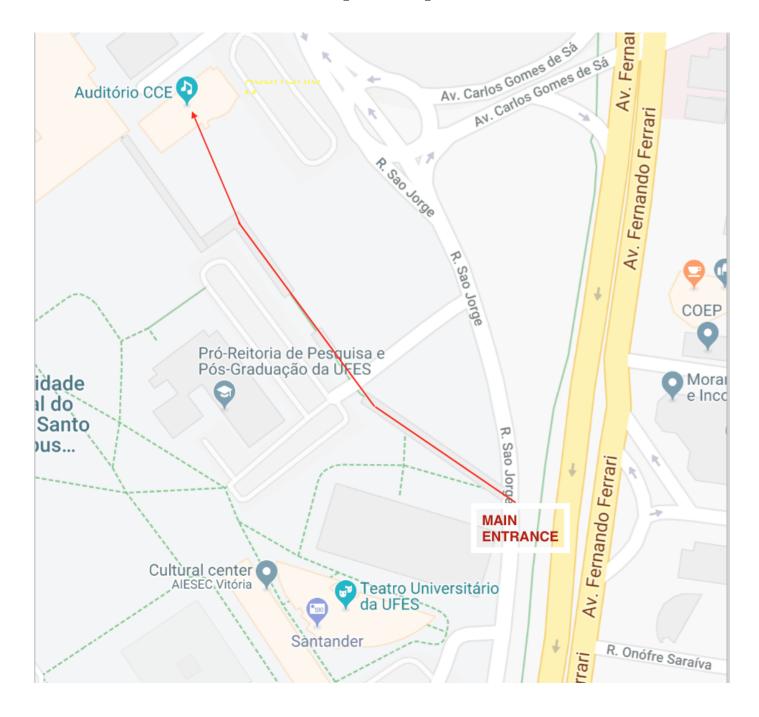
The present book is a collection of works to be presented during the meeting, as: mini-courses (8 hours lectures), plenary talks (50 minutes talks), communications talks (30 minutes) and posters presentations. Further details can be found in the link

https://mfsinges2019.wixsite.com/mfsinges2019

#### CONTENTS

Introduction		2
1.	List of Confirmed Participants	4
2.	Specials Plenaries Talks	7
3.	Mini-courses	8
4.	List of Talks in Foliation Theory	9
5.	Talks in Singularity Theory	11
6.	Talk in other area - Physics Mathematics	16
7.	List of Posters Presentations	16

# Campus Map



### 1. List of Confirmed Participants

It follows below in alphabetic order the list of all confirmed participants.

- (1) Abramo Hefez (UFF Brazil)
- (2) Adriana Laurindo Monteiro (UFES Brazil)
- (3) Alan Muniz (UFES Brazil)
- (4) Aldicio José Miranda (UFU Brazil)
- (5) Alexandre Fernandes (UFC Brazil)
- (6) Allan Ramos de Souza (UFMG Brazil)
- (7) Amanda Monteiro (UNESP Brazil)
- (8) Ana Claudia Nabarro (ICMC/USP Brazil)
- (9) André Belotto da Silva (Université Aix-Marseille France)
- (10) Aron Simis (UFPE Brazil)
- (11) Ayane Adelina da Silva (UFMG Brazil)
- (12) Braz Borges de Oliveira Filho (UFES Brazil)
- (13) Bruno Fonseca Coelho (CUSC Brazil)
- (14) Caio Adler Scalser Coimbra (UFES Brazil)
- (15) Carles Bivià Ausina (Universitat Politècnica de València Spain)
- (16) Crislaine Kuster (UFES Brazil)
- (17) Daiane Alice Henrique Ament (UFSCar Brazil)
- (18) Deyze Santos Carvalho (UFES Brazil)
- (19) Diogo Bessam (UFES Brazil)
- (20) Diogo da Silva Machado (UFV Brazil)
- (21) Diogo Taylor Ciciliotti (UFES Brazil)
- (22) Evandro Alves Nakajima (UTFPR Brazil)
- (23) Fábio Júlio Valentim (UFES Brazil)
- (24) Fernando Pereira Paulucio Reis (UFRJ Brazil)
- (25) Francisco Miguel Zamora Inuma (UFF Brazil)
- (26) Gabriel Nazarh Aprahamian de Oliveira Ramão (UFES Brazil)
- (27) Gilberto Cuzzuol (UNIFEI Brazil)
- (28) Gilcione Nonato Costa (UFMG Brazil)
- (29) Gilvan Borges Miranda (UFES Brazil)
- (30) Ginnara Mexia Souto (UFES Brazil)
- (31) Gustavo Franco Marra Domingues (UNIFEI Brazil)
- (32) Hana Marinho Lucena (UFES Brazil)
- (33) Hans-Christian Herbig (UFRJ Brazil)
- (34) Hellen Monção de Carvalho Santana (ICMC/USP Brazil)
- (35) Iris de Oliveira Zeli (ITA Brazil)

- (36) João Carlos Ferreira Costa (UNESP Brazil)
- (37) Joelso Giovanelli (UFES Brazil)
- (38) Jonny Ardila Ardila (Unila Brazil)
- (39) Jorge Vitório Pereira (IMPA Brazil)
- (40) José André Lourenço (UFES Brazil)
- (41) José Antonio Seade (UNAM Mexico)
- (42) Juan Viu Sos (ICMC/USP Brazil)
- (43) Julio Leo Fonseca Quispe (UFMG Brazil)
- (44) Karine Ramos Modesto (UFES Brazil)
- (45) Konstantinos Kourliouros (ICMC/USP Brazil)
- (46) Leandro Nery de Oliveira (UFAC Brazil)
- (47) Leonardo Meireles Câmara (UFES Brazil)
- (48) Lev Birbrair (UFC Brazil)
- (49) Lucas Arruda de Almeida (UFES Brazil)
- (50) Lucas Venâncio da Silva Santos (UESB Brazil)
- (51) Luis Guillermo Martinez Maza (UFAL Brazil)
- (52) Maico Felipe Silva Ribeiro (UFES Brazil)
- (53) Marcelo José Saia (ICMC/USP Brazil)
- (54) Marcio F Cerqueira (UFES Brazil)
- (55) Marcos Jardim (Unicamp Brazil)
- (56) Marcos Mercandeli Rodrigues (UFES Brazil)
- (57) Maria Aparecida Soares Ruas (ICMC/USP Brazil)
- (58) Maria Michalska (UFC Brazil)
- (59) Marlene Rodrigues Souza (UFES Brazil)
- (60) Marta Batoréo (UFES Brazil)
- (61) Massimo Ferrarotti (Politecnico di Torino Italy)
- (62) Mateus Schmidt Mattos Lopes Pereira (IME USP)
- (63) Miriam Garcia Manoel (ICMC/USP Brazil)
- (64) Misha Verbitsky (IMPA Brazil)
- (65) Nero Budur (KU Leuven Belgium)
- (66) Nguyen Thi Bich Thuy (Unesp Brazil)
- (67) Nhan Nguyen (ICMC/USP Brazil)
- (68) Nivaldo de Góes Grulha Júnior (ICMC/USP Brazil)
- (69) Olivier Thom (IMPA Brazil)
- (70) Patrícia Tempesta (UFSJ Brazil)
- (71) Paulo Murilo Brito Bomfim Santana (UFSCar Brazil)
- (72) Priscila dos Santos Brito (UFRB Brazil)
- (73) Rafaela Soares de Carvalho (UFSCar Brazil)
- (74) Raimundo N. Araújo dos Santos (ICMC/USP Brazil)

- (75) Renato Fehlberg Júnior (UFES Brazil)
- (76) Rogério Santos Mol (UFMG Brazil)
- (77) Sergey Agafonov (UNESP Brazil)
- (78) Terence James Gaffney (Northeastern University USA)
- (79) Thais Maria Dalbelo (UFSCar Brazil)
- (80) Thiago Fassarella do Amaral (UFF Brazil)
- (81) Thiago Filipe da Silva (UFES Brazil)
- (82) Tito Alexandro Medina Tejeda (ICMC/USP Brazil)
- (83) Valmecir Antonio dos Santos Bayer (UFES Brazil)
- (84) Vângellis Oliveira Sagnori Maia (UFMG Brazil)
- (85) Víctor Arturo Martínez León (UNILA Brazil)
- (86) Vincent Grandjean (UFC Brazil)
- (87) Wayner Moysés Marcelino (UFES Brazil)
- (88) Weverton Gomes Ribeiro (UFRB Brazil)
- (89) Wilker Thiago Resende Fernandes (UFSJ Brazil)
- (90) Yago Morais Zocoli (IFES Brazil)
- (91) Yuri Nuyryn Silva Mucci (UFES Brazil)

2. Specials Plenaries Talks

Below is the list of 4(four) specials plenaries speakers. They were initially invited to deliver a

special talks according to their mains contributions on its domains areas.

Singularities of codimension one foliations on projective threefolds

Jorge Vitório (IMPA - Brazil)

**Abstract:** I will present results on the global structure of codimension one foliations on projective

threefolds obtained through a semi-local study of neighborhoods of their singular sets. Emphasis

will be given on foliations carrying saddle-node singularities.

Morse-Bott foliations

José Antonio Seade (UNAM - Mexico)

**Abstract:** A smooth function  $f: M \to R$  on an n-manifold M is Morse-Bott if its critical points

are non-degenerate submanifolds. That is, the critical points are a disjoint union of submanifolds

 $N_1, \ldots, N_k$  an at each point  $x \in N_i$  one has that for each sufficiently small transversal for  $N_i$  at x,

f has a Morse critical point at x. A foliation in M is Morse-Bott if it can be defined locally by

Morse-Bott functions. In this talk we speak about Morse-Bott foliations and some of its properties.

Linear sections of determinantal varieties

Maria Aparecida Soares Ruas (ICMC-USP - Brazil)

Abstract: TBA

Locally trivial families in real analytic geometry

Misha Verbitsky (IMPA - Brazil)

**Abstract:** Let G be a group acting on a complex varieties X and B by automorphisms, and

 $f: X \mapsto B$  a proper, G-invariant map. Suppose that all orbits of G in B are dense. This implies

that the fibers of X are all homeomorphic and bi-Lipschitz equivalent. However, it is not proven

(and possibly even false) that these fibers are real analytically equivalent, if the singularities are

bad enough. I would explain how this situation arises (examples come from hyperkahler geometry)

and how it is possible to deal with it.

This is a joint work with Ekaterina Amerik

7

#### 3. Mini-courses

The 2 (two) mini-courses are planned to be a 4 (four) hours lectures each one, performing a total of 8 (eight) hours lectures. It will be offered to graduate students, post-docs, young and experienced researchers interested in working on common problems on the frontier of Foliations and Singularities theories.

## 1) Distributions and currents

Lecturer: Marcio Soares (UFMG - Brazil)

Abstract: TBA

## 2) Equisingularity and the theory of Integral Closure

Lecturer: Terence James Gaffney (Northeastern University - USA)

Abstract: This short course will study equisingularity, and the related ideas from commutative algebra - closure operations and multiplicities, which are bound up with it. A family of sets or mappings is called equisingular if all of the members of the family "are alike" in some way. Different notions of equisingularity arise from different meanings of "all alike".

Some of the notions of equisingularity we will study are: constant analytic type (all family members are equivalent by biholomorphic coordinate changes), constant topological type (equivalence by homeomorphisms) constant bi-Lipschitz type (bi-Lipschitz homeomorphisms) and Whitney equisingularity.

More details can be found in the link <a href="https://mfsinges2019.wixsite.com/mfsinges2019">https://mfsinges2019.wixsite.com/mfsinges2019</a>>

### 4. List of Talks in Foliation Theory

## Symmetries of foliations on the projective plane

Alan Muniz (UFES - Brazil)

**Abstract:** Let  $\mathcal{F}$  be a foliation on  $\mathbb{P}^2$  of degree  $d \geq 5$ . We will prove that if the automorphism group  $\operatorname{Aut}(\mathcal{F})$  is finite, then  $|\operatorname{Aut}(\mathcal{F})| \leq 6(d-1)^2$ . In order to achieve this we will classify every foliation such that the order of its automorphism group exceeds  $3(d^2+d+1)$ . This classification is obtained in two steps: the first one by analysing invariant loci for the group and the second one by establishing a Molien type formula that allows one describe foliations invariant by a given group. We will also show that  $|\operatorname{Aut}(\mathcal{F})| = 3(d^2+d+1)$  if and only if  $\mathcal{F}$  is the Jouanolou foliation. This is a joint work with Rudy Rosas.

## Strong Sard Conjecture in sub-Riemannian geometry

André Belotto da Silva (Université Aix-Marseille - France)

**Abstract:** Given a totally nonholonomic distribution of rank two  $\Delta$  on a three-dimensional manifold M, it is natural to investigate the size of the set of points  $\mathcal{X}^x$  that can be reached by singular horizontal paths starting from a same point  $x \in M$ . In this setting, the Sard conjecture states that  $\mathcal{X}^x$  should be a subset of the so-called Martinet surface of 2-dimensional Hausdorff measure zero.

In this seminar, I present a reformulation of the conjecture in terms of the singular behavior of a vector field. Next, I present a recent work in collaboration with Alessio Figalli, Ludovic Rifford and Adam Parusinski, where we show that the (strong version of the) conjecture holds in the analytic category and in dimension 3. Moreover, by studying the regularity of the solutions of the vector-field, we show that sub-Riemannian geodesics are all  $C^1$ . Our methods rely on resolution of singularities of surfaces, vector-fields and metrics; regularity analysis of Poincare transition maps; and a control of the divergence of vector fields.

# Second type foliations on $\mathbb{C}^3$

Gilberto Cuzzuol (Unifei - Brazil)

**Abstract:** We study a class of germs of codimension one holomorphic, non dicritical, singular foliations on  $\mathbb{C}^3$ , with no tangent saddle-nodes in their reduction of singularities. Such class of foliations are called *second type foliations* and has the property that the reduction of singularities of a second type foliation agrees with the reduction of its separatrix set.

# Local normal forms of singular Levi-flat hypersurfaces

Gustavo Franco Marra Domingues (Unifei - Brazil)

**Abstract:** We study normal forms of germs of singular real-analytic Levi-flat hypersurfaces. We prove the existence of rigid normal forms for singular Levi-flat hypersurfaces which are defined by the vanishing of the real part of complex quasi-homogeneous polynomials with isolated singularity. This result generalizes previous results of Burns-Gong and Fernández-Pérez. Furthermore, we prove the existence of two new rigid normal forms for singular real-analytic Levi-flat hypersurfaces which are preserved by a change of isochore coordinates, that is, a change of coordinates that preserves volume.

# Codimension one holomorphic distributions on $P^3$

Marcos Jardim (IMECC-UNICAMP - Brazil)

Abstract: We study codimension one holomorphic distributions on the projective three-space, analyzing the properties of their singular schemes and tangent sheaves. We describe the moduli space of distributions in terms of Grothendieck's Quot-scheme for the tangent bundle. In certain cases, we show that the moduli space of codimension one distributions on the projective space is an irreducible, nonsingular quasi-projective variety. Furthermore, we show that codimension one distributions of arbitrary degree with only isolated singularities have stable tangent sheaves, while every rational foliation and certain logarithmic foliations also have stable tangent sheaves. Finally, we provide a classification of codimension one distributions of degree at most 2 with locally free tangent sheaves. Joint work with O. Calvo-Andrade and M. Correa.

#### Pairs of Morse functions

Olivier Thom (France)

**Abstract:** I will present you the classification of pairs of Morse functions on  $\mathbb{C}^n$  locally in a neighborhood of 0. This problem is a particular case of the more general problem: classify the pairs of germs of functions (f,g) where f and g are known functions (the case where f and g are regular and transverse is trivial, and the case where f is regular and g is Morse is already known). As we will see, this particular case is already hard enough to give a good insight into the general problem.

#### Germs of Levi-flat foliations

Rogério Santos Mol (UFMG - Brazil)

**Abstract:** A singular real analytic foliation  $\mathcal{F}$  of real codimension one on an n-dimensional complex manifold M is Levi-flat if each of its leaves is foliated by immersed complex manifolds of dimension n?1. These complex manifolds are leaves of a singular real analytic foliation  $\mathcal{L}$  which is tangent to  $\mathcal{F}$ . We classify germs of Levi-flat foliations at  $(\mathcal{C}^n, 0)$  under the hypothesis that  $\mathcal{L}$  is

a germ of holomorphic foliation. Essentially, we prove that there are two possibilities for  $\mathcal{L}$ , from which the classification of  $\mathcal{F}$  derives: either it has a meromorphic first integral or is defined by a closed rational 1-form.

(Joint work with Arturo Fernández-Pérez (UFMG) and Rudy Rosas (PUCP))

# Irreducible Holonomy Groups and First Integrals for Holomorphic Foliations

Victor Arturo Martinez Leon (UNILA - Brazil)

Abstract: We study groups of germs of complex diffeomorphisms having a property called irreducibility. The notion is motivated by the similar property of the fundamental group of the complement of an irreducible hypersurface in the complex projective space. Natural examples of such groups of germ maps are given by holonomy groups and monodromy groups of integrable systems(foliations) under certain conditions. We prove some finiteness results for these groups extending previous results of D. Cerveau and F. Loray. Applications are given to the framework of germs of holomorphic foliations. We prove the existence of first integrals under certain irreducibility or more general conditions on the tangent cone of the foliation after a punctual blow-up. This is a joint work with M. Martelo and B. Scárdua.

## 5. Talks in Singularity Theory

# The Tjurina number for complete intersection curves

Abramo Hefez (UFF - Brazil)

**Abstract:** In this talk we address to the problem of finding the Tjurina number of a complete intersection curve with several branches. We give a recursive formula in general and a closed formula for curves with up to 3 branches.

This is a joint work with V. Bayer, E. Guzman and M. E. Hernandes.

# Singular 4-webs of asymptotic lines of spacelike surfaces in de Sitter 5-space

Ana Claudia Nabarro (ICMC/USP - Brazil)

**Abstract:** No nosso trabalho Second order geometry of spacelike surfaces in de Sitter 5-space, Publ. Mat. 59 (2015), investigamos propriedades de uma elipse especial de superfícies spacelike no espaço de Sitter  $S_1^5$ . Classificamos a segunda forma fundamental com respeito às direções normais lightlike e descrevemos as possíveis configurações desta elipse. Conseguimos informações sobre a superfície e caracterização de direções especiais sobre a superfície, como das direções binormais e

assintóticas. Mostramos que as direções assintóticas são dadas por uma equação diferencial (4-web) cujos coeficientes são escritos usando os coeficientes da segunda forma fundamental. Mostramos que as direções binormais são dadas também por uma equação de grau 4. Estas resultados são muito importantes para obter informações geométricas da superfície usando a teoria de contato e para o estudo de equações diferenciais que definem as linhas assintóticas. Neste novo trabalho que iremos apresentar (continuação do trabalho de 2015), estudamos as equações de direções assintóticas e das binormais destas superfícies e em especial os pontos singulares que podem aparecer para a superfície levantada dada pela equação binária das direções assintóticas. Surpreendentemente estas singularidades são não isoladas.

### Hankel like determinantal singularities

Aron Simis (UFPE - Brazil)

**Abstract:** Hankel matrices are overwhelmingly present in various parts of mathematics, perhaps mainly under the Toeplitz disguise. The goal is to study the corresponding determinantal singularities, including some degenerate situations. This is ongoing joint work with Rainelly Cunha (ITRN), Maral Mostafazadehfard (IMPA) and Zaqueu Ramos (UFS).

### Mixed Bruce-Roberts numbers

Carles Bivià-Ausina (Universitat Politècnica de València - Spain)

**Abstract:** We extend the notion of  $\mu^*$ -sequence and Tjurina number of functions to the framework of Bruce-Roberts numbers, that is, to pairs formed by the germ at 0 of a complex analytic variety  $X \subseteq \mathbb{C}^n$  and a finitely  $\mathcal{R}(X)$ -determined analytic function germ  $f:(\mathbb{C}^n,0)\to(\mathbb{C},0)$ . We analyze some fundamental properties of these numbers and relations between them. The structure of the module of tangent vector fields to a given variety  $X \subseteq \mathbb{C}^n$  plays a fundamental role in our study. This is a joint work with M.A.S. Ruas.

# $\mathcal{A}_e$ -codimension and image Milnor number for maps between curves

Daiane Alice Henrique Ament (UFSCar - Brazil)

**Abstract:** We define the image Milnor number of  $f:(X,0) \to (C^2,0)$ , where (X,0) is a space curve ICIS and f is a finite map germ of degree 1 onto its image (Y,0). If  $(X,0) \subset (C^n,0)$  is an irreducible weighted homogeneous singularity curve and  $f:(X,0) \to (C^2,0)$  is a map germ finite, one-to-one and weighted homogeneous with the same weights of (X,0). We show that  $\mathcal{A}_e$ -codim $(X,f) = \pi_I(f)$ , where  $\mathcal{A}_e$ -codim(X,f) is the  $\mathcal{A}_e$ -codimension, i.e., the minimum number of parameters in a versal deformation and  $\pi_I(f)$  is the image Milnor number, i.e., the number of vanishing cycles in the image of a stabilisation of f.

This is a joint work with J. J. Nuño Ballesteros and J. N. Tomazella.

## Symplectic quotients have symplectic singularities

Hans-Christian Herbig (UFRJ - Brazil)

Abstract: In the study of quadratic moment maps there appears to be a dichotomy between large and small representations. In the large case the moment map exhibits some features of regularity. This enables one to make qualitative and quantitative statements about the symplectic quotients (aka Hamiltonian reductions). In the talk I will report on recent joint work with Gerald Schwarz and Christopher Seaton which proves that for 2-large representations the appropriately defined complex symplectic quotient has symplectic singularities. This in particular entails that the symplectic quotient is graded Gorenstein domain and is a normal variety with rational singularities. It is expected that this result generalizes to small representations as well, even though here the moment map can have all sorts of pathologies. For this conjecture it is crucial to use the definition of the symplectic quotient that involves the real radical of the ideal generated by the moment map.

# The generic unfolding of a codimension-two connection to a two-fold singularity of planar Filippov systems

Iris de Oliveira Zeli (IEF-ITA - Brazil)

Abstract: Generic bifurcation theory was classically well developed for smooth differential systems, establishing results for k-parameter families of planar vector fields. In the present work we focus on a qualitative analysis of 2-parameter families,  $Z_{\alpha,\beta}$ , of planar Filippov systems assuming that  $Z_{0,0}$  presents a codimension-two minimal set. Such object, named elementary simple two-fold cycle, is characterized by a regular trajectory connecting a visible two-fold singularity to itself, for which the second derivative of the first return map is nonvanishing. We analyzed the codimension-two scenario through the exhibition of its bifurcation diagram.

(Co-autores: D.D. Novaes (IMECC/UNICAMP) e M.A. Teixeira (IMECC/UNICAMP)).

#### Referências

- [1] A.F. Filippov. Differential equations with discontinuous righthand sides: control systems, Springer Science & Business Media, 2013.
- [2] D.D. Novaes, M.A. Teixeira and I.O. Zeli. The generic unfolding of a codimension-two connection to a two-fold singularity of planar Filippov systems, *Nonlinearity* **31** (2018), n. 5, p. 2083.
- [3] M.A. Teixeira. Perturbation theory for non-smooth systems, *Encyclopedia of Complexity and Systems Science* **22** (2009), Springer, 6697–6719.
- [4] M.A. Teixeira. Non Smooth Dynamical Systems (NSDS): Reflections and Guidelines, 2017.

# A new formula for the motivic and topological zeta functions from $\mathbb{Q}$ -resolution of singularities

Juan Viu Sos (ICMC/USP - Brazil)

**Abstract:** We study motivic zeta functions for Q-divisors in a Q-Gorenstein variety. By providing a change of variables formula in this context, we give a closed formula for zeta functions of normal crossing divisors in the case where the ambient space contains quotient singularities given by finite abelian groups, providing specific examples. This result extends a previous result of W. Veys for the case of curves.

## Impasse Singularities of Constrained Hamiltonian Systems

Konstantinos Kourliouros (ICMC-USP - Brazil)

Abstract: A constrained Hamiltonian system is a Hamiltonian system restricted on a submanifold of the phase space representing the constraints. In this talk, motivated by an old problem of P. A. M. Dirac, I will present local classification results for all typical singularities of such systems at impasse points, where the Hamiltonian vector field is not well defined and/or smooth. The problem includes the classification of generic Euler-Lagrange equations as well as the classification of singular Lagrangians, i.e. of first order in the velocities.

## Some remarks on Lipschitz normal embeddings

Maria Michalska (UFC - Brazil)

**Abstract:** We will present connection between extensions of piecewise polynomial functions and Lipschitz normal embeddings. This gives rise to some necessary algebraic conditions for affine algebraic sets, real and complex, to be Lipschitz normal embedded.

# Local approximation of semianalytic and subanalytic sets

Massimo Ferrarotti (Politecnico di Torino - Italy)

**Abstract:** Two subanalytic subsets of  $\mathbb{R}^n$  are called s-equivalent at a common point P if the Hausdorff distance between their intersections with the sphere centered at P of radius r vanishes to order > s when r tends to 0. We proved that every s-equivalence class of a closed semianalytic set contains a semialgebraic representative of the same dimension. Results on approximation of subanalytic sets under suitable assumptions were obtained as well.

Joint work with E.Fortuna, L.Wilson.

# Symmetric pairs of planar foliations

Miriam Garcia Manoel (ICMC-USP - Brazil)

**Abstract:** In this talk we explain how to recognize symmetries in binary differential equations (BDEs). These are implicit differential equations given by the zeros of a quadratic 1-form,  $a(x,y)dy^2+b(x,y)dxdy+c(x,y)dx^2=0$ , for a,b,c smooth real functions defined on an open set of  $\mathbb{R}^2$ . Generically, solutions of a BDE are given as leaves of a pair of foliations, and the action of a symmetry must depend not only whether it preserves or inverts the plane orientation, but also whether it preserves or interchanges the foliations. We first present this dependence, which reveals to be given algebraically by a simple formula. We then present the general expressions of the symmetric quadratic 1-forms under all representations of the orthogonal group  $\mathbf{O}(2)$ .

This is a joint work with Patricia Tempesta, UFSJ - São João del Rei, MG.

# Groups and singularities

Nero Budur (KU Leuven - Belgium)

**Abstract:** The representations of the fundamental group of an algebraic variety form important topological invariants connecting group theory with geometry. In this talk, we focus on the varieties of smallest non-trivial dimension: curves. Using quivers and jet schemes, we show that the space of representations has at most rational singularities. We apply this to show that the number of complex representations of  $SL_n(Z)$  of dimension at most m grows at most as the square of m, for a fixed n > 2.

## On classification of Lipschitz A-simple function germs

Nhan Nguyen (ICMC/USP - Brazil)

**Abstract:** In a recent article we introduced the notion of Lipschitz R-simple function germs and presented a complete classification in the complex case. In this talk we will show that a germ is Lipschitz R-simple if and only if it is Lipschitz A-simple.

## The Nash modifications and the bi-Lipschitz equivalence

Nivaldo de Góes Grulha Júnior (ICMC-USP - Brazil)

**Abstract:** In this talk, we present partial results about relations between the Nash modification of two bi-Lipschitz equivalent germs of surfaces. It is a work in progress with Jean-Paul Brasselet, Alexandre Fernandes and Maria Ruas.

# A result on the 2-dimensional Complex Jacobian Conjecture under the view-point of "pertinent variables"

Nguyen Thi Bich Thuy (UNESP - Brazil)

**Abstract:** Let  $F: \mathbb{C}^2 \to \mathbb{C}^2$  be a polynomial mapping satisfying the Jacobian Condition of the Jacobian Conjecture. With the hypothesis the asymptotic set of F is not empty, we define new variables called pertinent variables and we treat T under these variables. That provides us a class of polynomial mappings which are not counter-examples for the 2-dimensional Complex Jacobian Conjecture.

#### TBA

Raimundo Nonato Araújo dos Santos (ICMC-USP - Brazil)

Abstract: TBA

# Contact structure on Fano variety of quadric and conservation laws of Hamiltonian PDEs

Sergey Agafonov (UNESP - Brazil)

**Abstract:** Systems of 3 PDEs of hydrodynamic type without Riemann invariants admit at most 5 conservation laws. There is a geometrically motivated natural way to associate such systems to 1-ruled 4-dimensional surfaces in  $P^6$ . It turnes out that an important class of such systems, namely Hamiltonian PDEs, is described in terms of contact structure on Fano variety of quadric.

# Re-parameterizing and reducing families of normal operators

Vincent Grandjean (UFC - Brazil)

Abstract: We present a new proof of (and extends) results of Kurdyka & Paunescu, and of Rainer, about real-analytic multi-parameters generalizations of classical results by Rellich and Kato about the

reduction in families of univariate deformations of normal operators over real or complex vector spaces of finite dimensions. Given a real analytic family of normal operators over a finite dimensional real or complex vector space, there exists a locally finite composition of blowings-up with smooth centers reparameterizing the given family such that at each point of the source space of the re-parameterizing mapping, there exists a neighbourhood of any given point over which exists a real analytic orthonormal frame in which the pull back of the operator is in reduced form at every point of the neighbourhood. Unlike all the previous demonstrations of such a result, we do not try to regularize the eigen values. A by-product of our approach is that we have, after reparameterization, that the eigen-values are analytic.

## 6. Talk in other area - Physics Mathematics

# $\textbf{Resurgence Analysis in Quantum Field Theory: Prolegomena} \text{-} \ \texttt{Physics/Mathematics}$

José André Lourenco (UFES - Brazil)

Abstract: Renormalized perturbation theory for QFTs typically produces divergent series, because the series coefficients grow factorially at high order. It has been a physical and mathematical challenge understand the asymptotic nature of perturbative series, and it has been unclear in what precise sense semiclassical expansions capture the physics of even weakly coupled QFTs. In this work we will discuss a recent conjecture that the semiclassical expansion of path integrals for asymptotically free QFTs yields well-defined answers once the implications of resurgence analysis are taken into account. Resurgence theory relates expansions around different saddle points of a path integral to each other, and has the striking practical implication that the high-order divergences of perturbative series encode precise information about the non-perturbative physics of a QFT.

### 7. List of Posters Presentations

# A presentation matrix associated to map germ from $(\mathbb{C}^n, 0)$ to $(\mathbb{C}^p, 0)$ , n < p

Aldicio José Miranda (UFU - Brazil)

**Abstract:** Given a finite map germ  $f:(\mathbb{C}^n,0)\to(\mathbb{C}^p,0), n< p$ , the objective is to present an algorithm and an implementation to compute a presentation matrix of the pushforward module  $f_*\mathcal{O}_{\mathbb{C}^n}$ . Some examples for the pairs (2,4) and (3,5) will be calculated.

Joint work with: M. E. Hernandes, O. N. Silva and G. Penafort-Sanchis.

# Holomorphic distributions on $\mathbb{P}^3$

Allan Ramos de Souza (UFMG - Brazil)

**Abstract:** A codimension one distribution  $\mathcal{F}$  of degree d on  $\mathcal{P}^3$  can be represented by a section  $\omega \in H^0(\omega_{\mathbb{P}^3}^1(d+2))$ , given by the dual of morphism  $\pi: T\mathcal{P}^3 \to N_{\mathcal{F}}$ , where  $N_{\mathcal{F}}$  is normal sheaf of  $\mathcal{F}$ . In this poster, we want to present the study of distributions whose singular scheme is a disjoint union, between an algebraic curve  $C \subset \mathcal{P}^3$  and isolated points. After a Blowing-up a long the curve C we want to compare numbers  $\sum Res(\omega, C)$  and  $\sum Res(\tilde{\omega}, \tilde{p_i})$  for a special distribution.

# Study of weighted homogeneous polynomials via Seifert forms

Amanda Monteiro (UNESP - Brazil)

**Abstract:** In this work we study nondegenerate weighted homogeneous polynomials by means of their weights. For this purpose, we study the Seifert form associated with a polynomial  $f: \mathbb{C}^{n+1} \to \mathbb{C}$ , which is a form on the Milnor fiber homology group associated with f, defined by linking number of two cycles.

We also define a polynomial  $P_f(t)$  that depends only on the weights of f. Then, we show the following result:

Theorem: Let f and g be a nondegenerate weighted homogeneous polynomials in  $\mathbb{C}^{n+1}$ . If their Seifert forms are equivalent over the real numbers then  $P_f(t) \equiv P_g(t) \mod (t^2 - 1)$ .

Saeki, in [1], also has proved that if  $P_f(t) \equiv P_g(t) \mod (t^2 - 1)$  then f and g have equivalent real Seifert forms. Therefore, exist a necessary and sufficient condition for these polynomials to have equivalent real Seifert forms in terms of their weights.

Advisers: Michelle Ferreira Zanchetta Morgardo and Évelin Meneguesso Barbaresco. Mathematics Department, Instituto de Biociências, Letras e Ciências Exatas - Unesp.

#### References:

- [1] O. Saeki, Real Seifert form determines the spectrum for semiquasihomogeneous hypersurface singularities in  $\mathbb{C}^3$ , J. Math. Soc. Japan 52 (2000), 409-431.
- [2] J. Milnor, Singular points of complex hypersurfaces, Princeton University Press, University of Tokyo Press, 1968.

## Hipersuperficies Levi-Flat e Webs Holomorfas

Ayane Adelina da Silva (UFMG - Brazil)

Abstract: Nosso objetivo é o estudo de variedades reais analíticas Levi-flat com singularidades em espaços complexos. Por definição, uma variedade real é Levi-flat se ela é folheada por subvariedades complexas. As variedades Levi-flat aparecem naturalmente como exemplos de subconjuntos invariantes de folheações holomorfas em variedades complexas. Primeiro, faremos uma descrição sobre a classificação local de folheações holomorfas com uma hipersuperfície Levi-flat invariante, seguindo os principais resultados de D. Cerveau e A. Lins Neto [CL]. Em continuação, serão apresentados problemas em aberto sobre a classificação de k-webs holomorfas singulares que deixam invariante uma variedade Levi-flat. Finalmente, serão apresentados alguns exemplos.

References

[CL] Cerveau, D., and Lins Neto, A.: Local Levi-flat hypersurfaces invariants by a codimension one holomorphic foliation. American Journal of Mathematics, vol. 133 no. 3, (2011), 677-716. doi.org/10.1353/ajm.2011.0018

# Global residue formula for logarithmic indices of foliations

Diogo da Silva Machado (UFV - Brazil)

Abstract: TBA

### Brasselet number and functions with one-dimensional critical set

Hellen Monção de Carvalho Santana (ICMC-USP - Brazil)

**Abstract:** Let  $f, g: (X, 0) \to (\mathbb{C}, 0)$  be germs of analytic functions defined over a complex analytic space X. The Brasselet number of a function f describes numerically the topological information of its generalized Milnor fibre. In this work, we present formulas to compare the Brasselet numbers of f, g and of the restriction of f to  $X \cap \{g = 0\}$ , in the case where g has a one-dimensional critical locus.

# Topological classification of circle-valued simple Morse-Bott functions

João Carlos Ferreira Costa (UNESP - Brazil)

**Abstract:** In this work we investigate the classification of Morse-Bott functions from  $S^2$  to  $S^1$ , up to topological conjugacy. We give a complete topological invariant of simple Morse-Bott functions  $f: S^2 \to S^1$ . The invariant is based on the generalized Reeb graph associated to f (called here MB-Reeb graph). Moreover, a realization theorem is obtained.

This is a joint work with E.B. Batista and I.S. Meza-Sarmiento.

## Invariant polynomials by the action of a Lie group of even index

Leandro Nery de Oliveira (UFAC - Brazil)

**Abstract:** We present a algorithm to calculate a Hilbert basis of the ring of the invariant polynomials of a  $\Gamma$  group, since we know the Hilbert basis of the ring of the invariant polynomials of a subgroup  $\Sigma$  of  $\Gamma$  of even index, using the Reynolds operators.

## Symmetric homogeneous binary differential equations

Patrícia Tempesta (UFSJ - Brazil)

**Abstract:** Homogeneous binary differential equation are implicit differential equations of the form  $a(x,y)dy^2 + 2b(x,y)dxdy + c(x,y)dx^2 = 0$ , where the coefficients a,b,c are polynomial functions of degree n. This special class of binary differential equations has the property that the symmetry group is always no trivial, and furthermore, we can see that the invariant straight lines that can occur have different behavior depending on parity of the degree of the coefficients.

Co-author: M. Manoel.

# $\mu$ -constant deformations of functions on ICIS

Rafaela Soares de Carvalho (UFSCar - Brazil)

**Abstract:** Greuel, in 1986, studied properties of the constancy of the Milnor number of a deformation  $f_t: (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$  of a holomorphic function germ with isolated singularity resulting in the following theorem:

Theorem: Let  $f:(\mathbb{C}^n,0)\to(\mathbb{C},0)$  be a holomorphic function germ with isolated singularity at the origin. For any deformation  $F:(\mathbb{C}^n\times\mathbb{C})\to(\mathbb{C},0)$  of f the following statements are equivalent:

- (1) F is  $\mu$ -constant.
- (2) For every holomorphic curve  $\gamma: (\mathbb{C},0) \to (\mathbb{C}^n \times \mathbb{C},0)$

$$\nu\left(\frac{\partial F}{\partial t}\circ\gamma\right) > \inf\{\nu\left(\frac{\partial F}{\partial x_i}\circ\gamma\right) \mid i=1,\ldots,n\},\,$$

(where  $\nu$  denotes the usual valuation of a complex curve).

- (3) Same statement as in (2) with ">" replaced by " $\geq$ ".
- (4)  $\frac{\partial F}{\partial t} \in \overline{J_F}$ , (where  $\overline{J_F}$  denotes the integral closure of the Jacobian ideal  $J_F = \left\langle \frac{\partial F}{\partial x_1}, \dots, \frac{\partial F}{\partial x_n} \right\rangle$  in  $\mathcal{O}_{n+1}$ ).
  - (5)  $\frac{\partial F}{\partial t} \in \sqrt{J_F}$ , (where  $\sqrt{J_F}$  denotes the radical of  $J_F$ ).
  - (6)  $v(J_F) = \{0\} \times \mathbb{C} \text{ near } (0,0).$

In this work, we studied this result for families  $f_t:(X,0)\to(\mathbb{C},0)$ , where (X,0) is an ICIS.

Joint work with Bruna Oréfice Okamoto and João Nivaldo Tomazella. Mathematics Department, Universidade Federal de São Carlos - UFSCar.

### Brasselet number and Newton polygons

Thaís Maria Dalbelo (UFCar - Brazil)

**Abstract:** We present a formula to compute the Brasselet number of  $f:(Y,0) \to (\mathbb{C},0)$  where  $Y \subset X$  is a non-degenerate complete intersection in a toric variety X. As applications we establish several results concerning about invariance of the Brasselet number for families of non-degenerate complete intersections. Moreover, when  $(X,0) = (\mathbb{C}^n,0)$  we derive sufficient conditions to obtain the invariance of the Euler obstruction for families of complete intersections with isolated singularity which are contained on X.

#### The Fundamental Theorem for Frontal Surfaces

Tito Alexandro Medina Tejeda (ICMC/USP - Brazil)

**Abstract:** In recent years, there is a great interest in the study of the geometry of a special class of singular surfaces, namely, Frontals. A map-germ f defined in the euclidean two-space to the euclidean three-space is called a Frontal if there exists a unit normal vector field v along f. This means, v is ortogonal to the partial derivatives of f. In this work we discuss the existence of a fundamental theorem for this class of singular surfaces. The results are part of my PhD Thesis supervised by Maria Aparecida Ruas.

#### TBA

Vângellis Oliveira Sagnori Maia (UFMG - Brazil)

Abstract: TBA

## Some properties of the May-Leonard asymmetric system

Wilker Thiago Resende Fernandes (UFSJ - Brazil)

Abstract: The main objective of this poster is to present some results on the existence of first integrals for the three dimensional May-Leonard asymmetric system. Using the computational algebra systems Mathematica and Singular we first look for families of the May-Leonard asymmetric system admitting invariant surfaces of degree two. Then using these invariant surfaces and invariant planes we construct first integrals of the Darboux type identifying subfamilies of such system admitting one first integral and two independent first integrals. This is a joint work with Valery Antonov (Peter the Great St.Petersburg Polytechnic University, St.Petersburg, Russia), Valery Romanovski (Center for Applied Mathematics and Theoretical Physics, Maribor, Slovenia) and Natalie Shcheglova (Belarusian State University, Minsk, Belarus).

References

- [1] V. Antonov, D. Dolićanin, V. G. Romanovski, J. Tóth. Invariant planes and periodic oscillations in the May-Leonard asymmetric model. *MATCH Commun. Math. Comput. Chem.*, **76** (2016) 455–474.
- [2] R. M. May, W. J. Leonard. Nonlinear aspects of competition between three species. SIAM J. Appl. Math., 29 (1975) 243–253.
- [3] V.G. Romanovski and D.S. Shafer, The Center and cyclicity Problems: A computational Algebra Approach. Boston: Birkhauser, 2009.